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Practical Implications of Pole Series Convergence and the Early-time in Transient Backscatter

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14. ABSTRACT

Here we consider the convergence of the singularity expansion method (SEM) pole series representation of the transient backscatter in terms of the stability in the estimated poles from the time domain response. Using a numerically simulated backscattered response of a thin wire, the temporal progression of the estimated pole series is examined by iteratively estimating them (using the Matrix Pencil Method) at different time-window locations. The results show a good agreement with the well established SEM theory (i.e., a stable set of poles are obtained after the theoretically defined time for convergence). This iterative approach could allow for a more accurate estimation of the poles that represent the resonances of the object. We also consider the use of the "non-converging" poles estimated in the early-time to approximately represent the early-time specular response.

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CONTENTS

ABSTRACT

It is well known that transient backscatter response in the resonance region could provide useful information regarding the natural resonances of the scattering object, and such parameters are often referred to as the singularity expansion method (SEM) parameters (i.e. poles and residues). Poles and residues, which respectively represent the object's complex natural frequencies and their corresponding amplitudes, can be estimated from the late-time response that is represented as a sum of damped sinusoids (pole series expansion in the Laplace domain). Since the transient backscatter also consists of the early-time response, which occurs before the late-time, it cannot be represented only with the damped sinusoids. In the mathematical framework of the SEM, this means that the pole series converges only in the late-time. Here we consider the convergence of the pole series representation of the transient backscatter in terms of the stability in the estimated poles from the time domain response. Using a numerically simulated backscattered response of a thin wire, the temporal progression of the estimated pole series is examined by iteratively estimating them (using the Matrix Pencil Method) at different time-window locations. The results show a good agreement with the well established SEM theory (i.e. a stable set of poles are obtained after the theoretically defined time for convergence). This iterative approach could allow for a more accurate estimation of the poles that represent the resonances of the object. We also consider the use of the "non-converging" poles estimated in the early-time to approximately represent the early-time specular response.

PRACTICAL IMPLICATIONS OF POLE SERIES CONVERGENCE AND THE EARLY-TIME IN TRANSIENT BACKSCATTER

INTRODUCTION

According to the singularity expansion method (SEM), the transient backscattered response of an object can be represented in terms of two components, namely the early-time and the late-time responses [1]. The early-time response is a forced response generated while the incident pulse is still interacting with the object, and generally contains specular reflections of the incident pulse from different scattering centers of the object as well as the forced oscillation due to the traveling wave on the object [2]. The late-time response is generated after the incident pulse has passed the target where there remains only the induced current oscillating at the natural resonance frequencies of the object, and can be represented as a sum of damping sinusoids (complex exponentials), in which the complex frequencies correspond to the natural resonances. Figure 1 describes the transient backscatter response of an object in free space.

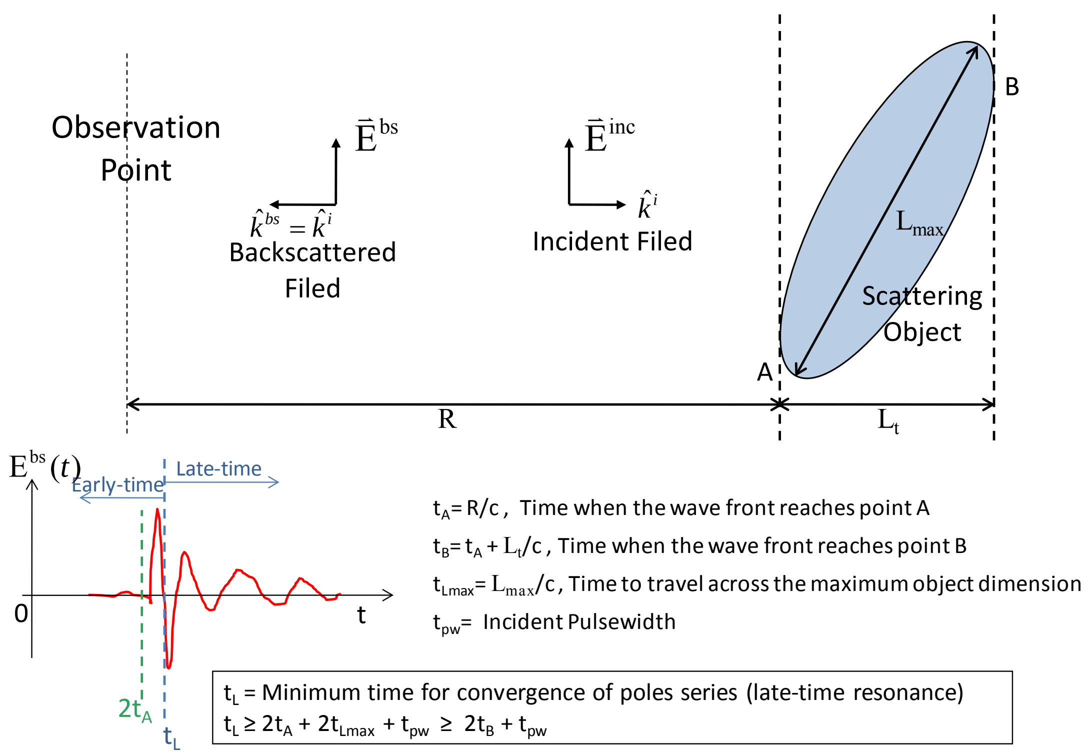

Figure 1. Transient backscatter response of a finite-size object in free space and the definition of t_L, which determines the early-time and late-time boundary. Waveforms launched and observed from the observation point.

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The time-domain SEM representation of the transient backscattered response of an object is

$$\mathrm{E}^{\mathrm{bs}}(\hat{k}^i,\hat{e}^i,\hat{e}^s,t)=\mathrm{E}_{\mathrm{e}}(\hat{k}^i,\hat{e}^i,\hat{e}^s,t)u\left(t_L-t\right)+\sum_n R_n(\hat{k}^i,\hat{e}^i,\hat{e}^s)e^{s_n(t-t_L)}u\left(t-t_L\right)\ , \tag{1}$$

where the first term is the early-time response and the second terms is the late-time response. Here *u(t)* is the Heaviside step function. In the late-time term, $s_n=\sigma_n+j2\pi f_n$ are the poles[1] that correspond to the natural frequencies of the object, with σ_n and f_n respectively representing the damping constants and the oscillating frequencies, and R_n are the residues, which are the complex amplitudes of the corresponding poles. In the Laplace domain, equation (1) is represented as

$$\tilde{\mathrm{E}}^{\mathrm{bs}}(\hat{k}^i,\hat{e}^i,\hat{e}^s,s)=\tilde{\mathrm{E}}_{\mathrm{e}}(\hat{k}^i,\hat{e}^i,\hat{e}^s,s)e^{-st_L}+e^{-st_L}\sum_n\frac{R_n(\hat{k}^i,\hat{e}^i,\hat{e}^s)}{s-s_n}e^{s_nt_L}\ , \tag{2}$$

where $\tilde{\ }$ denotes the Laplace transform. The early-time term in the Laplace domain is represented by an entire function, and the late-time term is represented by the expansion of the poles. The poles are aspect-independent, that is, they are determined only by the physical properties of the target (shape, size, and material property). The residues, however, are aspect-dependent (direction of incidence, polarization of incidence and polarization of scattering, respectively shown as $\hat{k}^i,\hat{e}^i$ and $\hat{e}^s$) and related to the scattering pattern of the object. Equations (1) and (2) corresponds to the class-1 form of the SEM representation, in which the residue values are time-independent [3]. It should also be noted that a function continuous at t_L, (rapidly decays afterwards) should better represent the early-time response. However, the "gated" early-time should be sufficient for the purpose of describing the early/late-time boundary and the late-time pole series representation.

Due to the aspect-independent poles, which can be used as distinctive features of a scattering object, the late-time response has been of particular interest in applications such as scatterer characterization [4] and target identification [5]. From the time-domain backscattered response the poles can be extracted by fitting the signal as a sum of complex exponentials [6,7]. For an accurate pole estimation in practical applications, a time window of the sampled backscattered response must be in the late-time (after t_L, see Figure 1), where the signal can be represented only by the second term in Equation (1). In the theoretical framework of the SEM, this means that the pole series solution is guaranteed to converge in the late-time [3]. If the time window starts before t_L (i.e. the windowed signal cannot be represented only by the convergent sum of decaying sinusoids), the estimated poles may also include the ones that do not contribute to the convergent pole series, hereafter referred to as the non-converging poles.

In this paper, the pole series convergence is considered from a practical perspective in terms of the stability in the estimated SEM parameters from the time-domain backscattered response. A numerically simulated backscattered response of a thin wire is used as an example. Convergence is examined by estimating the poles at different locations of the time window in a consecutive manner, which provides a temporal progression of the pole series representation of the response. This approach allows for more accurate estimation of the "true" poles (converging poles that correspond to the natural resonances), without requiring a priori knowledge of t_L, since they can be determined from the time progression of the

[1] This SEM representation assumes simple, first order poles only.

estimated poles. A similar approach was used in the past to determine the resonant frequencies of buried mines [8], but was not investigated in much details. Also considered in this paper, is the use of the non-converging early time poles to represent the "entire function" corresponding to the early-time response, and its theoretical relevance. This could provide useful information in terms of the impulsive specular scattering part of the response, thus providing possible additional features for target characterization and identification.

SIMULATED THIN WIRE BACKSCATTER

The transient backscattered response of a thin wire model is simulated using SEMCAD, a finite difference time domain computer code [9]. In the model a 15cm long thin wire was assumed to be perfectly conducting and the backscattered response was obtained at 2.1 meters of distance by exciting a plane wave of 130-ps Gaussian[2] pulse. As indicated in Figure 2, backscatter responses from two different wire orientations, i.e. $\theta = 0^\circ$ and 45°, are used. The computational domain is truncated with absorbing boundary condition. The simulated responses are shown in Figure 3.

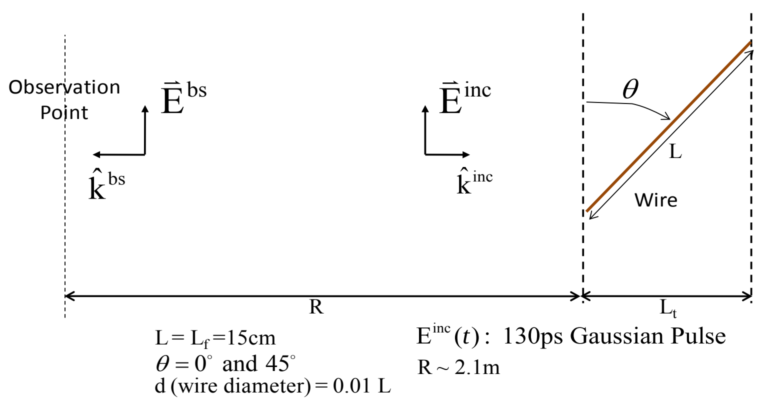
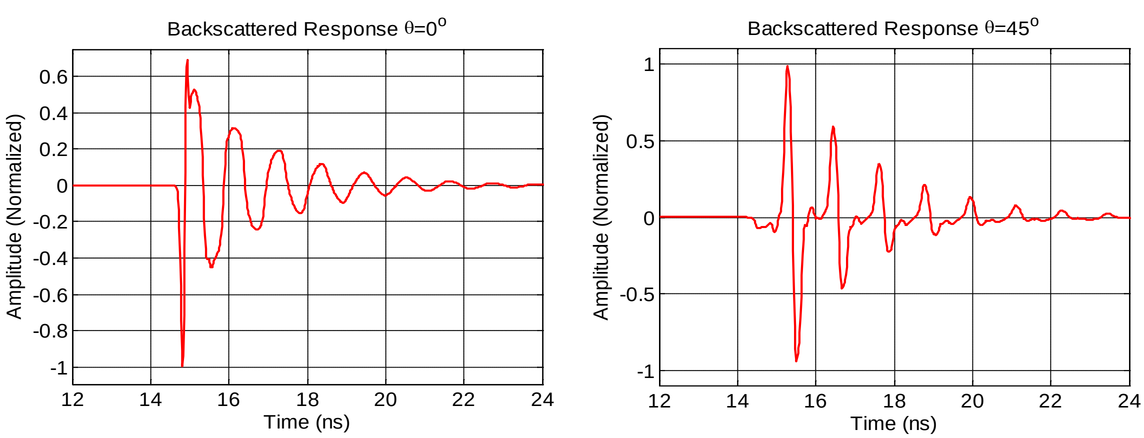


Figure 2. Thin wire simulation setup. The incident field is vertically polarized ($\theta = 0^\circ$), and the wire is parallel to the plane of incidence. This configuration allows the backscattered field to be co-polarized with the incident field.



a) $\theta = 0^\circ$ b) $\theta = 45^\circ$

Figure 3. Simulated backscattered thin wire responses

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[2] It is practically impossible to radiate a Gaussian pulse due to its DC component. However, it is used in the simulation to yield a response that is close to the true impulse response of the object.

CONVERGENCE OF ESTIMATED POLE SERIES

The poles and the corresponding residues are estimated using the matrix pencil method (MPM), which has been known for its computational efficiency and effectiveness in the presence of noise[7,10]. In this process, time-windowed samples of the response are represented in terms of the sample index m and the sampling interval Δ_t as

$$\mathrm{E}^{\mathrm{bs}}\left(\mathrm{m}\Delta_t\right) \approx \sum_{n=1}^{N} R_n z^{\mathrm{m}} \text{ , for m=1,...,M}, \tag{3}$$

where $z = e^{s_n \Delta_t}$, M is the number of samples in the time-windowed signal, and N is the maximum number of desired poles to estimate. The poles are estimated by rearranging the samples into sequential matrices, and then solving a generalized eigenvalue problem. Once the poles are found, the residues can be obtained using a least squares approach.

Convergence of the pole series is examined by iteratively estimating the location of the SEM parameters in the Laplace domain at different time window locations as shown in Figure 4. The width of the window (i.e. $t_{\mathrm{f}} - t_o$) was chosen to be 5ns so as to include at least a few periods of the fundamental resonance. The initial and final t_o of the iterative process were 14.4ns and 17.8ns, respectively. A total of 34 different sets of pole series were estimated with the increment of 0.1ns. In Figure 5, a temporal progression of the estimated poles (f_n only) are shown as a function of t_o on the top of the original response (left), and the complex poles are shown in the Laplace domain (right). In both cases, it is shown that the poles converge to a stable set of locations after t_L, which shows a good agreement with the SEM framework, where t_L is defined as the minimum time for the pole series convergence [3]. It is also observed that only the symmetric mode resonances are extracted for $\theta = 0^{\circ}$, whereas both symmetric and non-symmetric modes are extracted for $\theta = 45^{\circ}$. The non-converging poles that are extracted before t_L vary in time and do not represent the natural resonances of the scatterer, since their values are the results of attempt to fit the early-time signal to the late-time representation. Some of these poles may be the result of the MPM's attempt to estimate the poles in the second layer [1], which are very difficult to excite and estimate. Figure 6 also shows the temporal progression of the poles in terms of their corresponding residue amplitudes. In the figure, it is clearly seen that the residues of the non-converging poles vary in time more than that of the converging poles while that of the converging poles are relatively constant.

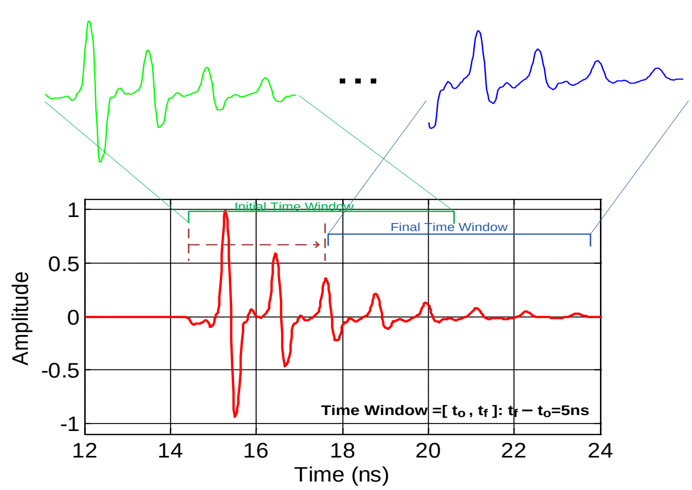
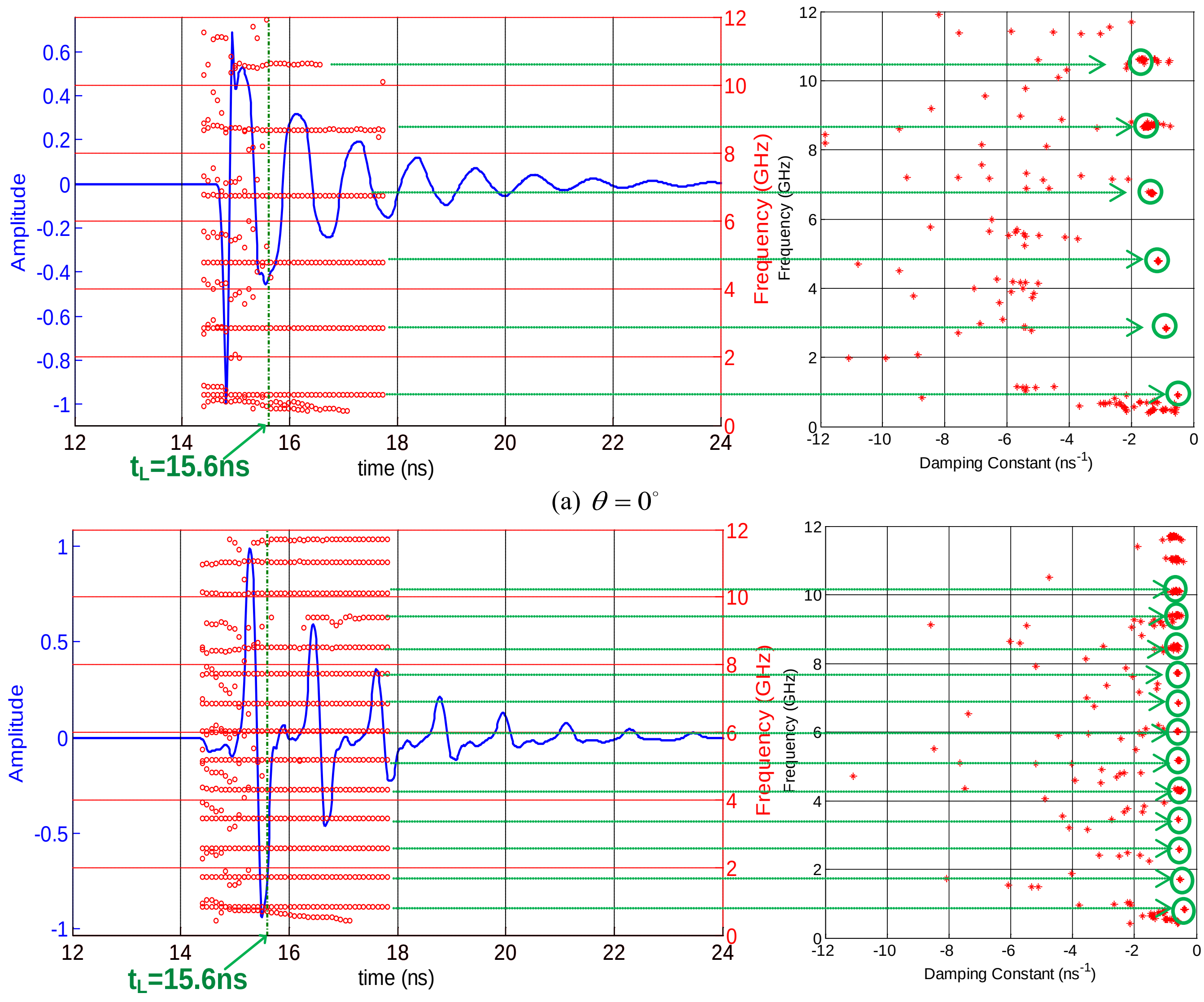


Figure 4. Iterative pole extraction approach by sliding the sample time window



(a) $\theta = 0^{\circ}$

(b) $\theta = 45^{\circ}$

Figure 5. Estimated poles from the wire backscatter responses. The plots on the left show the poles (frequencies only, no damping constants) estimated at corresponding time window starting points as the window is shifted. The plots on the right show the same poles in the Laplace domain (damping constant vs. frequency). Convergent poles show stability in their values with time window shift.

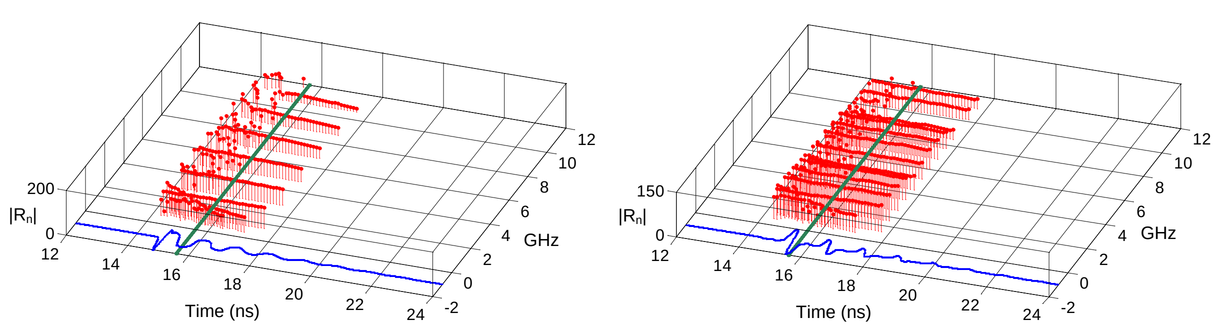


Figure 6. Estimated poles (frequencies only, no damping constants) plotted in terms of corresponding residue amplitudes, $|R_n|$. (log scale)

Since the converging poles can be clearly determined from the temporal progression in Figure 5, this approach could be an effective way to accurately estimate resonant parameters without requiring an initial guess for the early-time/late-time boundary, especially if the scatterer is unknown.

REPRESENTATION OF THE EARLY-TIME USING ESTIMATED POLE SERIES

Another useful insight is made regarding the non-converging poles when t_o is in the early-time (before t_L). Some additional target characteristics may be examining the non-converging poles. Figure 7 shows the estimated poles for $\theta = 0^\circ$ when $t_o = 14.74$ns. From the temporal progression plot in Figure 5, it is possible to distinguish the converging poles from non-converging ones based on the consistency of their locations in the Laplace domain as shown in Figure 7. Therefore this pole series can be separated into two different sums, namely *np* series and *p* series, corresponding to the non-converging and converging poles, respectively as follows:

$$\mathrm{E}_{\mathrm{r}}^{\mathrm{bs}}\left(t; t_o = 14.74\mathrm{ns}\right) = \sum_{np} R_{np} e^{s_{np}(t-t_o)} u(t-t_o) + \sum_{p} R_p e^{s_p(t-t_o)} u(t-t_o). \tag{4}$$

Using equation (4), the response is reconstructed into two different signals as shown in Figure 8. The reconstructed signal from *p* series corresponds to the slowly-varying part in the early-time and continues into the late-time, closely matching the original response. The signal reconstructed from *np* series (dashed green line) is more impulsive than that reconstructed from *p* series (solid blue line) and vanishes after t_L, which indicates that *np* series contributes to the early-time forced response.

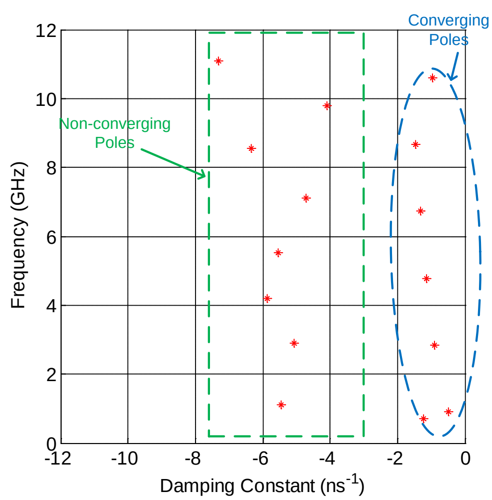


Figure 7. Estimated Poles for $\theta = 0^\circ$ when t_o=14.74ns. Using the data from Figure 5a, the converging and non-converging poles can be determined.

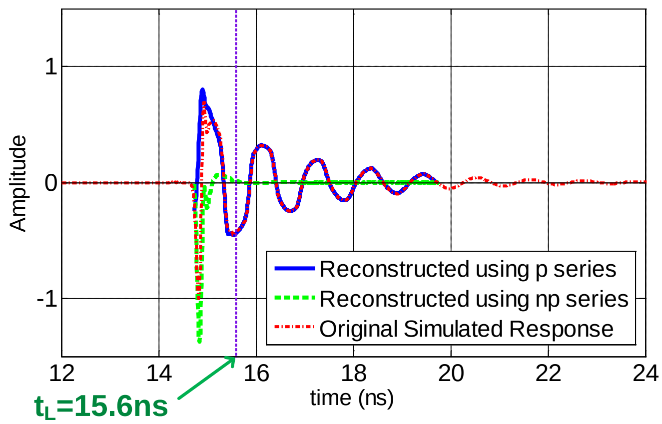


Figure 8. Reconstructed signals from two separate pole series (estimated using t_o=14.74ns): converging poles (solid blue), non-converging poles (dashed green). Original response is shown in dotted red.

In the Laplace domain equation (4) can be expressed as[3]

$$\tilde{E}_r^{bs}(s) = \sum_{np} \frac{R_{np}}{s - s_{np}} + \sum_{p} \frac{R_p}{s - s_p} \ . \tag{5}$$

According to the SEM framework, the early-time representation should be an entire function, which means that the first term in equation (5) is not a valid representation. However, we can relate the term to the generalized representation of an entire function in the left-half plane using the Cauchy integral [11], which is

$$\tilde{x}_{entire}(s) = \frac{1}{2\pi j} \int_{c_\infty} \frac{R(s')}{s - s'} ds' \ , \quad \text{where } c_\infty = \lim_{\Delta \to \infty} c \text{ (left-plane contour).} \tag{6}$$

The Cauchy integral in equation (6) can be considered as poles with continuous residue distribution $R(s')$ at $s \to \infty$, suggesting that the entire function can be regarded as a function with the singularities at infinity. Therefore, from this perspective the pole (or singularities) in the first term of equation (5) can be approximately represented as an "entire function" to describe the early-time forced response, which could provide useful information in the impulsive part of the response, thus providing possible additional features for target characterization and identification.

[3] Shifted left in time by t_o for the simplicity of the expression without loss of generality

SUMMARY AND CONCLUSION

In this paper, the convergence of the SEM pole series in transient backscatter was explored in terms of the stability in the estimated SEM parameters. Using the simulated response of a thin wire, the pole series convergence was investigated through an iterative approach that provided the temporal progression of the estimated poles. The analysis showed good agreement with the SEM theory, i.e. the pole series convergence was observed after the theoretically defined t_L, the minimum time for convergence. The iterative pole extraction approach was also shown to be an effective method of accurately estimating SEM parameters, particularly when the scatterer is unknown.

It was also suggested that the use of the non-converging early-time poles could approximately represent an “entire function” to describe the early-time forced response. This response could provide additional useful information about an unknown scattering object, e.g. the features extracted from the early-time forced response could be used to better characterize or identify unknown scatterers.

This report only considered a simple object in a noiseless environment (simulated). It would be of an interest to investigate various types of objects in more complex environment (e.g. noisy responses, multiple scatters, etc.) for more practical examples.

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